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Reg. No. : ....

ode No. : 30617 E   Sub. Code : CAMA 21

CBCS) DEGREE EXAMINATION, APRIL 2022.

Second Semester

Mathematics — Allied

CTOR CALCULUS AND FOURIER SERIES

or those who joined in July 2021 onwards)

Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

hoose the correct answer :

vector f is called solenoidal if ————.

) $\operatorname{div} \bar{f} = 0$ (b) $\operatorname{grad} \bar{f} = 0$

) $\operatorname{div} \bar{f} = 1$ (d) $\operatorname{curl} \bar{f} = 0$

f $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, then $\nabla \times \bar{r} =$ ————.

) $\bar{0}$ (b) 2

) 1   (d) $x^2 + y^2 + z^2$

3. If R is a rectangular region with vertices (0, 0), (2, 0), (2, 3) and (0, 3) then the value of $\iint_R dx\,dy =$ ————.

(a) 5   (b) 4

(c) 9   (d) 6

4. $\int_0^a \int_0^b \int_0^c dx\,dy\,dz =$ ————.

(a) $a+b+c$ (b) $a^3+b^3+c^3$

(c) abc (d) $a^3b^3c^3$

5. The value of $\int_0^\pi \int_0^1 r^4 \sin\theta\, dr\, d\theta$ is ————.

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) 1

6. If V is the volume enclosed by the closed surface S, then the value of $\iint_S \bar{r} \cdot \bar{n}\, dS =$ ————.

(a) $3V^2$ (b) $3V$

(c) $6V$ (d) 0

reen's theorem connects ————.

) line integral and double integral

) line integral and surface integral

) double integral and surface integral

) surface integral and volume integral

S is any closed surface enclosing a volume V nd $\bar{f} = ax\bar{i} + by\bar{j} + cz\bar{k}$, then $\iint_S \bar{f} \cdot \bar{n}\, dS =$ ————.

) $3V$ (b) $(a+b+c)V$

) $(a+b+c)^3 V^3$ (d) 0

$f(x)$ is an even function, ————.

) $f(x) = f(x^2)$ (b) $f(x) - f(x^2)$

) $f(x) = f(-x)$ (d) $f(x) = -f(-x)$

m is an integer, $\int_0^\pi \cos mx\, dx =$ ————.

) 1   (b) π

) $\frac{\pi}{2}$ (d) 0

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\operatorname{curl}(\operatorname{curl} \bar{f}) = \operatorname{grad} \operatorname{div} \bar{f} - \nabla^2 \bar{f}$.

Or

(b) Prove that $\operatorname{div}\left(\frac{\bar{r}}{r}\right) = \frac{2}{r}$.

12. (a) Find the area of the circle $x^2 + y^2 = r^2$ by using double integral.

Or

(b) Evaluate $\int_0^\pi \int_0^{a\cos\theta} r \sin\theta\, dr\, d\theta$.

13. (a) Find the work done by the force $\bar{F} = 3xy\bar{i} - 5z\bar{j} + 10x\bar{k}$ alone the curve C, $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

Or

(b) Evaluate $\iint_S \bar{f}\cdot\bar{n}\,dS$ where $\bar{f}=(x+y^2)\bar{i}-2x\bar{j}+2yz\bar{k}$ and S is the surface of the plane $2x+y+2z=6$ in the first octant.

14. (a) Verify Green's theorem for the function $\bar{f}=(x^2+y^2)\bar{i}-2xy\bar{j}$ and C is the rectangle in the xy plane bounded by $y=0$, $y=b$, $x=0$ and $x=a$.

Or

(b) Evaluate $\iint_S f.\bar{n}\,ds$ using Gauss divergence theorem for the vector function $\bar{f}=(x^3-yz)\bar{i}-2x^2y\bar{j}+2\bar{k}$ over the cube bounded by $x=0, y=0, z=0$, $x=a, y=a$ and $z=a$.

15. (a) Find half range cosines series for the function $f(x)=x^2$ in $(0,\pi)$

Or

(b) Express $f(x)=x$ $(-\pi<x<\pi)$ as a fourier series with period 2π.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) If $\bar{r}$ is the position vector of any point $P(x,y,z)$, prove that grad $r^n=nr^{n-2}\bar{r}$.

(ii) Find the unit normal to the surface $x^3-xyz+z^3=1$ at (1,1,1)

Or

(b) (i) Prove that :
grad $(\varphi\psi)=\varphi grad\psi+\psi+grad\varphi$ and

(ii) grad $\left(\dfrac{\varphi}{\psi}\right)=\dfrac{\psi\, grad\varphi-\varphi grad\psi}{\psi^2}$.

17. (a) Evaluate $\iint_D x^2y^2\,dx\,dy$ where D is the circular disc $x^2+y^2\le 1$.

Or

(b) Evaluate the following :

(i) $\int_0^a\int_0^x\int_0^y xyz\,dz\,dy\,dx$

(ii) $\int_0^{\pi}\int_0^{\pi/2}\int_0^1 r^2\sin\theta dr\,d\theta\,d\varphi$

18. (a) If $\bar{f}=(2y+3)\bar{i}+xz\,j+(yz-x)\bar{k}$, evaluate $\int_C \bar{f}.\,d\bar{r}$ along the following paths C

(i) $x=2t^2$, $y=t$, $z=t^3$ from $t=0$ to $t=1$

(ii) The polygonal path P consisting of the three line segments AB, BC and CD where $A=(0,0,0), B=(0,0,1), C=(0,1,1)$ and $D=(2,1,1)$

(iii) The straight line joining $(0,0,0)$ and $(2,1,1)$.

Or

(b) If $\bar{f}=(x^2-y^2)\bar{i}+2xy\bar{j}$, evaluate $\int_C \bar{f}.\,d\bar{r}$ along the curve C is the xy- plane given by $y=x^2-x$ from the point $(1,0)$ to $(2,2)$.

19. (a) Verify Gauss divergence theorem for $\bar{f}=y\bar{i}+x\bar{j}+z^2\bar{k}$ for the cylindrical region S given by $x^2+y^2=a^2$, $z=0$ and $z=h$.

Or

(b) Verify Stoke's theorem for $\bar{f}=(2x-y)\bar{i}-yz^2-y^2z\bar{k}$ where S is the upper half of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

20. (a) Find a cosine series in the range 0 to π for

$$f(x)=\begin{cases}x, & 0<x<\pi/2\\ \pi-x, & \pi/2<x<\pi\end{cases}.$$

Or

(b) Expand $f(x)$ in $(-\pi,\pi)$ as a fourier series if

$$f(x)=\begin{cases}-x, & -\pi<x<0\\ x, & 0\le x<\pi\end{cases}$$ and deduce

$$\frac{\pi^2}{8}=1+1/3^2+1/5^2+1/7^2+\ldots$$